



A data-driven analysis and optimization of the impact of prescribed fire programs on wildfire risk in different regions of the USA

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Abstract

In the current century, wildfires have shown an increasing trend, causing a huge amount of direct and indirect losses in society. Different methods and efforts have been employed to reduce the frequency and intensity of the damages, one of which is implementing prescribed fires. Previous works have established that prescribed fires are effective at reducing the damage caused by wildfires. However, the actual impact of prescribed fire programs is dependent on factors such as where and when prescribed fires are conducted. In this paper, we propose a novel data-driven model studying the impact of prescribed fire as a mitigation technique for wildfires to minimize the total costs and losses. This is applied to states in the USA to perform a comparative analysis of the impact of prescribed fires from 2003 to 2017 and to identify the optimal scale of the impactful prescribed fire programs using least-cost optimization. The fifty US states are classified into categories based on impact and risk levels. Measures that could be taken to improve different prescribed fire programs are discussed. Our results show that California and Oregon are the only severe-risk US states to conduct prescribed fire programs that are impactful at reducing wildfire risks, while other southeastern states such as Florida maintain fire-healthy ecosystems with very extensive prescribed fire programs. Our study suggests that states that have impactful prescribed fire programs (like California) should increase their scale of operation, while states that burn prescribed fires with no impact (like Nevada) should change the way prescribed burning is planned and conducted.

Keywords Decision analysis · Wildfires · Prescribed fire · Fuel management · Least-cost optimization

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1 Introduction

The Federal Emergency Management Agency (FEMA) defines a wildfire as “an unplanned, unwanted fire burning in a natural area, such as a forest, grassland, or prairie” (Federal Emergency Management Agency 2022). Wildfires are designated as “natural disasters” by the U.S. Environmental Protection Agency (EPA) (U.S. Environmental Protection Agency 2020) that have increasingly affected human society (Agarwal et al. 2020), built environment (for example, damage to public and private infrastructure), and ecosystem services (i.e., air quality and long-term carbon storage) around the world over the past two decades (Moritz et al. 2014). For example, the November 2018 Camp Fire that occurred in California was recognized then as the second-costliest wildfire in the history of the USA, with estimated losses of \$8.47 billion. The fire burned 153,300 acres, destroyed 18,804 structures, and resulted in 85 fatalities. In 2021, there were 7400 fire incidents in California that burned an estimated 2.5 million acres, damaged/destroyed 3629 structures, and caused 3 fatalities. There was a significant decrease in acreage burned in 2022, with 7700 incidents burning a little over 360,000 acres (CALFIRE 2022). While California has shown some recent progress, there is still much cause for concern nationally and globally. In Europe in 2022, double the average acreage of fire was burned in a record-setting season for both wildfires and heat (Phys.org 2022). A few years before that, in late 2019, record-breaking bushfires (wildfires) plagued Australia, where, in addition to deaths and devastation, Attiya and Jones (2022) found that everyday life was significantly disrupted by the air pollution caused by the fires.

Past studies have analyzed seasonal patterns in wildfire occurrences to understand the dynamics of changing wildfire regimes across the world. Bajocco et al. (2010) analyzed the importance of human activity and climatic characteristics of the landscape on wildfire seasonality in Mediterranean regions. The authors identified an early arrival of the fire season in urban and agricultural areas with favorable climatic conditions in comparison with forests, grasslands, and shrublands. Coogan et al. (2020) studied spatial and seasonal differences between lightning and human-caused wildfires in Canada. The authors found that human-caused wildfires are more common in the southern parts of the country with higher population density and anthropogenic activity. In contrast, the northern parts of the country have a higher occurrence of lightning-caused fires due to lower population density and anthropogenic activity. In terms of seasonality, lightning-caused fires occur more frequently during the summer months (June to August), while human-caused fires peak during the spring (in the month of May). Westerling et al. (2003) also found a strong influence of climate on wildfires in the contiguous western USA, where 94% of fires that accounted for 98% of the total area burned occurred between May and October. It was observed that the wildfire season peaks between July and August which represent the warmest and driest months in the entire year. The authors also investigated the importance of using the Palmer Drought Severity Index (PDSI) (Keeley and Syphard 2021), a measure of combined precipitation, evapotranspiration, and soil moisture conditions, as a basis for forecasting the severity of wildfire season months to years in advance. According to the study, such a forecast can help guide fuel management practices and resource allocation decisions.

Human action can not only mitigate the effects of wildfires but can intervene to prevent its onset and spread by using different strategies such as burning forests and grasslands proactively to prevent uncontrolled burning and clearing dry fuel from forest areas to prevent fires from starting (Ferris 2013). Different methods and efforts can be employed to reduce the frequency and intensity of the damages caused by wildfires, one of which is conducting

prescribed fires (also known as prescribed burns, controlled burns, scheduled burns, hazard reduction burns, burn-offs, planned ignitions, or backfires). Prescribed burning has a long history in the USA where indigenous people used fire for various cultural and ecological purposes. A prescribed fire (which we refer to as Rx in this paper) is a fire ignited and managed in a specific region for various reasons. These reasons include: to support diverse ecosystems, to reduce the fuel load of a region to make it less susceptible to wildfire losses, and even to protect wildland from a potential pyro-terror attack (large-scale arson) (Rashidi et al. 2018). Rx is an effective way to reduce the risks posed by wildfires, including deaths and injuries caused directly by burns and indirectly by smoke inhalation, damage to personal, public, and commercial properties and infrastructure, and disruption in the functioning of various industries (i.e., tourism, forestry, and transportation). While Rx is on the rise as a prevention and mitigation strategy for wildfires, it is often impeded by lack of funding, preparation, and guidance (Gass 2006).

The National Interagency Fire Center (NIFC) in collaboration with the National Interagency Coordination Center (NICC) publishes annual fire statistics. Up to 2017, the data are categorized into Rx and wildfires by state and agency. After 2018, the published data do not break down Rx by state or agency (National Interagency Fire Center 2022). Figure 1b shows that the density of wildfires from 2003 to 2017 is higher in the western states of the mainland USA and Alaska compared to the eastern part of the country. Here, density is defined as acres per 100 acres of forest cover as described by the Forest Inventory and Analysis Database (FIADB) (USDA Forest Service 2022). Winston and Shao (2022) have found that the western part of the USA has a “Mediterranean Climate” due to which it gets most of its rain during winter. As a result, the summer months are dry and hot which creates suitable conditions for wildfire ignitions. Additionally, the vegetation in the western region contains pine needles, dry grasses, and shrubs that are generally more flammable than the vegetation found in the deciduous forests of the East coast. Fires are more prevalent in states like California due to ideal conditions for a raging wildfire, such as dry and hot weather which dehydrates the large amount of fuel present in forests. From 2003 to 2018, the acres of wildfires in the USA doubled, from 4 million acres to 8.5 million acres. In the same time period, the number of wildfires in the USA decreased from 63,000 to 58,000, indicating that the average area burned by a wildfire has been on the rise, as

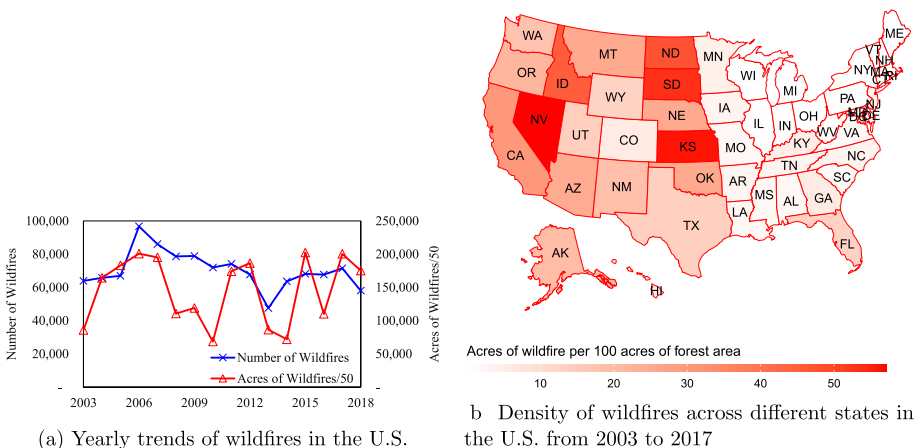


Fig. 1 Yearly trends (2003–2018) and density of wildfires in the USA (2003–2017)

observed in Fig. 1a. There have been 196 fires that spanned over 100,000 acres in the USA from 2001 to 2019. In August 2020, the August Complex fire in California (the largest complex fire in California history) started as 38 separate fires and burned over 1 million acres in less than three months, about 1% of California’s 100 million acres of land (U.S. Forest Service 2020). Furthermore, one of the fires of the complex, namely Doe Fire, is the biggest fire by area in California’s history, having reached close to half a million acres, while two other fires of the complex are in the third and fourth place for largest fires by area in California’s history (Neilson 2020).

Over 40 million acres of Rx have been conducted in the USA since 2003, with the number conducted exponentially increased by approximately 2500% from 2014 (17,044) to 2018 (450,335), as observed in Fig. 2a. Figure 2b shows the density of Rx use across the different states from 2003 to 2017. It is evident from this figure that southeastern states such as Florida, Georgia, Alabama, and South Carolina have a much higher density of Rx usage as compared to the other states. Florida runs one of the most active Rx programs in the country, overseen by the Florida Forest Service, with an average of 88,000 authorizations to burn more than 2 million acres a year (Florida Forest Service 2022). In 2017, that number hit a record high with over 4 million acres of Rx.

On the other hand, California has historically resisted prescribed burning to control their already big smoke problem (Peterson 2018). A research study recently discovered, however, that the smoke from wildfires is about 3 times more polluting than the smoke from Rx (Liu et al. 2017). Tools such as remote sensing, geographical information system, and statistical methods can be used to effectively detect and monitor smoke generated from wildfires and Rx (Abdollahi et al. 2021). A 129-page report by the Little Hoover Commission in early 2018 lobbied for more Rx in California and Nevada (which, combined, had more than 1.3 million acres of wildfire in 2017) to better manage the forests prevent overcrowding and hence disastrous fires, specifically in the Sierra Nevada mountains (Little Hoover Commission 2018). There are various reasons preventing California from conducting more Rx, including lack of enough firefighters and 57% of land being owned by the Federal Government which lacks the funds to conduct Rx, especially after recent budget cuts to the U.S. Forest Service (Rainey 2018; United States Department of Agriculture 2017). In spite of this, CAL FIRE, the California Department of Forestry and Fire

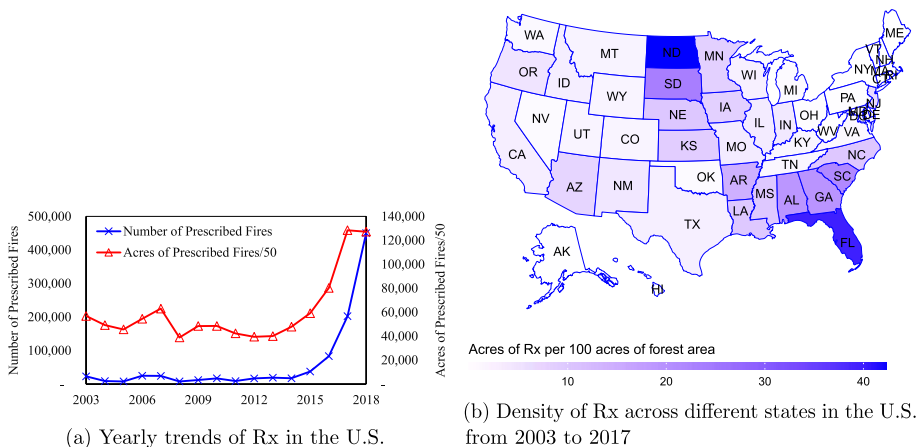


Fig. 2 Yearly trends (2003–2018) and density of Rx in the USA (2003–2017)

authors also studied the variation in burn windows over time that was found to shift from early spring to late fall.

Past researchers have studied the effectiveness of Rx on mitigating wildfires using various methods including case studies in a particular area (Martinson and Omi 2008), statistical assessments of fuel treatments in certain regions (Safford et al. 2012), simulation (Omi et al. 1999), and surveys (Kobziar et al. 2015). Various optimization methods have been applied to Rx and other fuel management techniques, such as (i) spatial optimization to determine location of fuel treatments given budgetary and other constraints (Alcasena et al. 2018; Palaiologou et al. 2020), (ii) optimization for balancing Rx and wildlife prevention activities (Butry et al. 2010), and (iii) multi-objective models that investigate the effects of different fuel treatment patterns on wildfires (Kim et al. 2009). These works have clearly established the effectiveness of Rx in mitigating wildfires and used various optimization models to identify the best Rx strategies. In spite of this, many regions do not use Rx or use them in smaller quantities than needed. To the best of our knowledge, the impact of large-scale Rx programs on wildfire risk has not been rigorously studied. Furthermore, the Rx programs of different states have not been quantitatively compared. Such comparative assessments play a vital role in assisting policy makers and communities in improving their respective regions' resilience (Kumar and Mehany 2022; Torkayesh et al. 2022). In this paper, we aim to answer the following research questions: **RQ1**: How impactful are Rx programs at mitigating wildfire risks in different US states? and **RQ2**: Do impactful Rx programs need to be scaled up, and by how much? To address these questions, we use a data-driven approach to study the impact of Rx in mitigating the losses due to wildfires. We also develop a least-cost optimization model to determine the optimal scale of Rx programs in different regions of the USA.

The rest of the paper is organized as follows: Sect. 2 discusses past works on the study of effectiveness of fuel management and optimization of the same. The research gaps and their addressal are expounded. Section 3 describes the exponential regression used to estimate the impact of Rx and the least-cost optimization model. Section 4 explains the data sources and parameter estimation used. Section 5 compares the impact of Rx programs across US states, where each US state is classified into different categories based on risk of wildfires and impact of Rx programs. The impact of Rx in Oregon is detailed, and the optimal costs and losses of Oregon in the context of wildfires across different years are compared. Section 6 presents recommendations for policy makers and practitioners based on the results. Section 7 summarizes our work and discusses its limitations and future extensions. "Appendix" provides the proofs for Propositions 1, 2 and 3 as given in Sect. 3.

2 Literature review

The effectiveness of Rx is widely studied. The effects of Rx on wildfires in specific treated areas or forests have been studied extensively. For example, in central Idaho, Arkle et al. (2012) examined the impact of Rx on the burn severity of wildfires post fuel treatment. The authors discovered that patchy, low-severity Rx are effective and the size, shape, and continuity of Rx may be more vital than the severity of the Rx. Martinson and Omi (2008) observed lower scorch height (average height of foliage browning), crown damage (damage from crown fires, where fire spreads from treetop to treetop), and ground char (a qualitative measure of damage to the soil) in Rx-treated area in Jackson County, Mississippi. Omi and Martinson (2002) also emphasized the importance of reduction in crown fuels which

outweighs an increase in surface fire hazard that may be stimulated by a prescribed burn. Safford et al. (2012) used statistical assessments to analyze the effectiveness of mechanical fuel thinning and prescribed burns in Eastern and Southern California. Treatments that include surface and ladder fuel removal were found to be consistently effective at reducing wildfire severity and increasing the chances of adult tree survival. Some of these studies are conducted using long-term data, such as Boer et al. (2009) who studied a 52-year fire history from the Eucalypt forest regions in southwestern Australia and found that a unit area reduction in wildfire requires about 4 unit areas of prescribed burning. In this study, the average annual extent of Rx over a period of 6 years explained a 24% variation in the average annual number and a 71% variation in the average annual extent of unintended wildfires. In a similar study in Portugal, Davim et al. (2022) estimated a past fire (both wildfire and Rx) leverage ratio of 0.28. That is, for every acre of past fire, subsequent wildfires are decreased by 0.28 acres. The estimated leverage ratio was estimated to be higher in places with higher wildfire occurrence rates.

While forests in different parts of the world burn and react differently to fuel treatment, studies all around the world indicate that Rx are effective at reducing wildfire risk in many different aspects. In a study based in North-Western Portugal, Fernandes and Botelho (2004) discovered that Rx reduce the potential intensity of an extreme-weather wildfire by 96%. They also found that only 36% of the treated area requires heavy suppression resources. Penman et al. (2020) studied the cost-effectiveness of Rx across various landscapes in southeastern Australia using a Phoenix RapidFire simulator. They discovered that cost-effectiveness varies widely between regions, landscape types, and proximity to urban interface. Price (2012) simulated the effects of fuel treatments on leverage (the negative impact of fuel treatment on unintended wildfires). The percentage of area burnt by these wildfires were found to have an exponential decay relationship with respect to the percentage of area treated. Stratton (2004) discovered that fuel treatment decreases size, spot fires, surface flame length, fireline intensity, and crown fire under 75th, 85th, and 95th percentile weather conditions.

In addition to establishing the effectiveness of Rx, there is extensive literature that optimizes Rx to increase its impact. Research has been done to determine the optimal location and layout of Rx. Alcasena et al. (2018) used spatial optimization to maximize the contribution being made to different competing objectives with the Landscape Treatment Designer. They ranked the fuel treatment priorities for landscapes in the western USA by considering constraints related to budget, implementation, and legislation. Palaiologou et al. (2020) used FlamMap Fire Simulation System to design optimal locations for fuel treatments. FlamMap identifies the fastest fire travel routes and then uses fuel treatments to block the spread of fires. They found that in Greece, fuel treatments in Conifer forests (particularly with Olive-Conifer connectivity) appear to be very effective. Butry et al. (2010) developed an optimization model for balancing the allocation of wildlife prevention activities (WPE) and Rx over space and time. The model determines the optimal amount of Rx and WPE in four regions in Florida taking into account variations due to weather, community factors, etc. Elia et al. (2016) maximized the cost-effectiveness of Rx using fire behavior simulations to determine WUI regions where reducing fuel load is the most cost-effective. Minas et al. (2015) maximized the number of “cells” or areas in a region that are suitably covered by deployed resources. The model was applied to 20 hypothetical 100-cell landscapes with hypothetical parameters. Vega-Garcia et al. (2014) optimized Rx planning specifically for the Mediterranean region using custom fuel models. Three study areas were selected, and their fuel loads and characteristics were measured before and after Rx to identify the impact of Rx on different fuel loads.

Other features of Rx have also been optimized, such as patterns of Rx. Finney (2004) developed an optimization model to minimize the fire growth rate by identifying spatially optimal fuel treatment unit sizes and locations. The authors used a genetic algorithm in combination with FARSITE to simulate fire growth across a given landscape. Their study shows that spatial patterns of fire treatments are critical to fire growth rates and randomly-arranged fire treatment units are very inefficient in changing overall fire growth rates. Mercer et al. (2007), on the other hand, identified the optimal rate of prescribed burning that minimizes the net economic losses from wildfires using Monte Carlo simulations. Konoshima et al. (2010) built a spatial optimization model to study the dynamics of harvesting and fuel treatment across a hypothetical landscape divided into 7 management units with 8 different spatial configurations. León et al. (2018) used mixed integer programming (MIP) to break the connectivity between high-risk regions while also taking into account the habitat quality of the burn unit. Matsypura et al. (2018) developed an MIP framework in which they used graph theoretical models to incorporate the directions of fire spread between adjacent cells. The constraints in the model included the fuel load and age of each cell and budget restrictions. Wei et al. (2008) developed a fuel treatment spatial allocation framework to model the fire risk of a cell as a function of the ignition probability within a cell and the conditional probability of external wildfire spreading to the cell.

Given that the effectiveness of Rx is widely established, and there are frameworks available to help make better Rx decisions, Rx is not conducted as much as it should be for a wide variety of reasons. Additionally, while Rx is effective, we want to study the large-scale impact of Rx programs as they vary based on how, where and when Rx is conducted. Koehler (1993) observed that decreasing the threat of wildfires is particularly vital in Wildland-Urban Interface (WUI) regions. However, landowners oppose Rx in these areas due to perceived danger to personal land and property. Even when decision-makers are convinced that Rx is beneficial, they are often constrained by inadequate funding, smoke management, air quality regulations, the necessity to comply with environmental protection, and liability risks (Fernandes and Botelho 2003). Kobziar et al. (2015) identified certain similar factors by conducting a survey in the southern USA, of 523 public and private fire practitioners. The major hurdle to prescribed burning is a lack of budget and staffing according to public land managers and liability according to private entities.

Given this disparity and the increasing worldwide risk of wildfires, it is vital that decision-makers understand the impact of Rx that is currently conducted and identify the regions that conduct Rx inefficiently. Furthermore, to the best of our knowledge, past studies have not provided a comparative analysis of the Rx conducted by region in order to identify areas where overhaul is required. To address these gaps, we create an analytical framework to classify regions into 4 categories based on risk and impact: (i) regions with high wildfire risk and inefficient Rx programs—these regions need better Rx strategies; (ii) regions with high wildfire risk and well-established large-scale Rx programs for multiple objectives including mitigating wildfire risk; (iii) regions with high wildfire risk and impactful Rx programs—these regions need to scale up their Rx programs; and (iv) regions with low wildfire risk. For the third category, we also use a least-cost optimization model to determine the optimal scale of the existing Rx program. Our models use data analytics to derive the functions that describe the wildfire risk of a region and the impact of Rx in mitigating these wildfires.

3 Notations and model

Table 1 provides descriptions of the notations for the decision variables, parameters, and functions used in the mathematical formulation of the least-cost model in this paper. In this paper, we measure the impact of Rx in reducing the number of acres of wildfires in a given region. The acres of wildfires, $z(x)$ as a function of the acres of Rx, x are defined as follows:

$$z(x) = \zeta(x)F \tag{1}$$

where $\zeta(x)$ is the vulnerability of a region to wildfires that is expressed as a function of x and F is the total acres of forest cover in a given region. The measure $\zeta(x)$ takes into account the burnable area of a given region. We assume that (i) $\zeta(x)$ is continuous and twice differentiable for $x \geq 0$ and (ii) $\zeta(x) > 0$, $\zeta'(x) < 0$, and $\zeta''(x) > 0$ for $x \geq 0$. We present a least-cost optimization model in which the best value of the decision variable (here, x) is found such that the total costs, losses, and other monetary expenses are minimized (Abhyankar et al. 2022; Anderson 1972). Our least-cost model optimizes the investment in Rx in a region by minimizing the total costs and losses associated with wildfires and Rx, $U(x)$, which is expressed as the sum of costs and losses due to wildfire in a given region and the cost of conducting Rx in said region as shown below:

$$\min_x U(x) = \underbrace{c_w z(x)}_{\text{costs and losses due to wildfire}} + \underbrace{c_p x}_{\text{cost of conducting Rx}} = c_w \zeta(x)F + c_p x \tag{2}$$

where c_w represents the costs and losses of wildfires per acre and c_p is the cost to conduct an acre of Rx. The c_p value for a region is calculated as a weighted sum:

$$c_p = \frac{\sum_j A_j c_{pj}}{\sum_j A_j} \times l \tag{3}$$

Table 1 Notations used throughout the paper

<i>Decision variables</i>	
$x \geq 0$	Acres of Rx conducted in a region
x^*	Optimal acres of Rx conducted in a region
<i>Parameters</i>	
$z > 0$	Acres of wildfires in a region
$F > 0$	Acres of forest cover in a region
$a > 0$	Base vulnerability of a region to wildfires
$\lambda > 0$	Baseline impact of Rx in a region
$c_p > 0$	Cost to conduct an acre of Rx
$c_w > 0$	Costs and losses per acre of wildfire
<i>Functions</i>	
$\zeta(x)$	Vulnerability of a region to wildfires
$\mathcal{G}(x)$	First derivative of $\zeta(x)$
$U(x)$	Total costs and losses due to wildfires and Rx
$U(x^*)$	Optimal total costs and losses due to wildfires and Rx

where c_{pj} is the cost per acre of conducting a certain j type of Rx (for example, slash reduction or natural Rx) in the given region, A_j is the acres of type j Rx conducted in the region, and t is used to adjust the value for inflation. c_w for a region is calculated as $\frac{c_{wr}}{z}$, where c_{wr} is the sum total of all estimated losses in a region due to wildfires and z is the acres of wildfires in that region. As a part of our work, we do not convert non-economic losses into economic losses. Instead, we use reports published by reputable sources that estimate the economic losses of non-economic wildfire damage. Further details on how c_w and c_p are calculated are given in Sect. 4.

Proposition 1 For a general $\zeta(x)$ that satisfies the aforementioned assumptions, the optimal amount of Rx to conduct (x^*) and the total optimal costs and losses of wildfires (U^*) are given as follows:

$$x^* = \begin{cases} \mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right) & \text{if } c_p < -\mathcal{G}(0)c_w F \\ 0 & \text{if } c_p \geq -\mathcal{G}(0)c_w F \end{cases} \tag{4}$$

$$U^* = U(x^*) = \begin{cases} c_w \zeta\left(\mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right)\right)F + c_p \mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right) & \text{if } c_p < -\mathcal{G}(0)c_w F \\ \zeta(0)c_w F & \text{if } c_p \geq -\mathcal{G}(0)c_w F \end{cases} \tag{5}$$

where $\mathcal{G}(x) = \zeta'(x)$.

Remark In the above proposition, the minimum value of U , U^* , and the corresponding x^* is calculated using differential calculus (Hancock 1917). The calculations are presented in detail in ‘‘Appendix.’’ Proposition 1 expresses x^* and U^* as functions of parameters c_p , c_w , and F , and vulnerability function $\zeta(x)$. The values of x^* and U^* are dependent on the relationship between c_p and $-\mathcal{G}(0)c_w F$. If $c_p < -\mathcal{G}(0)c_w F$, it is economically beneficial to conduct a positive number of Rx to minimize the losses and costs associated with wildfires. If $c_p \geq -\mathcal{G}(0)c_w F$, conducting Rx does not appear to be an economically viable option for decreasing wildfire risks and losses.

Studies such as Stratton (2004) observed an exponential decay relationship of the vulnerability of an area to wildfires with respect to the percentage of the area treated using prescribed burns. An exponential function was also used by Behrendt et al. (2019) to model the relationship between vulnerability to structural fire losses and investment in fire prevention. Past studies have also leveraged exponential functions to model the following important aspects of wildfires: (a) Petrovic et al. (2012) used exponential distribution to model the temporal dynamics of wildfire spread in terms of a stochastic birth–death process, (b) Alexandridis et al. (2008) used exponential function to study the effect of wind speed and ground elevation on the rate of wildfire spread, (c) Schoenberg et al. (2003) used exponential function to model the distribution of wildfire size in terms of the burn area, and (d) Sakellariou et al. (2020) used exponential function to model the frequency distribution of wildfire ignitions. Based on past models, we hypothesize the relationship between acres of wildfires and acres of Rx to be of the exponential decay form. Let λ be the baseline impact on wildfires per acre of Rx in a given region and a be the base vulnerability of a given region when $x = 0$, then $\zeta(x)$ is expressed as the following:

$$\zeta(x) = ae^{-\lambda x} \tag{6}$$

We assume that the Rx burning season occurs before the wildfire season (that is, Rx is usually conducted in the spring) in the USA, as supported by data like Missouri’s prescribed burn council burn log (Missouri Prescribed Fire Council 2021), except for a few western states such as California and Oregon where most Rx burning happens in the fall, after the wildfire season (Oregon Department of Forestry 2022). In the former, the impact of the current year’s Rx on the current year’s wildfires is studied, while in the latter case, the impact of the previous year’s Rx on the current year’s wildfires is studied. In Eq. 6, a higher λ value denotes a higher impact of the prescribed burns conducted in a region leading to a higher rate of decrease of $\zeta(x)$ with respect to x , as illustrated in Fig. 4.

Proposition 2 For a $\zeta(x)$ that is expressed in Eq. 6, the optimal acres of impactful Rx to conduct (x^*) and the total optimal costs and losses of wildfires (U^*) are given as follows:

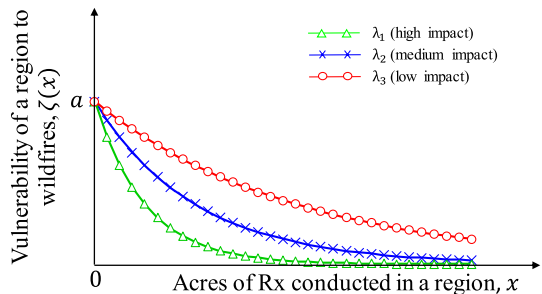
$$x^* = \begin{cases} -\frac{1}{\lambda} \ln\left(\frac{c_p}{c_w a F \lambda}\right) & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{7}$$

$$U^* = U(x^*) = \begin{cases} \frac{c_p}{\lambda} \left(1 - \ln\left(\frac{c_p}{c_w a F \lambda}\right)\right) & \text{if } c_p < c_w a F \lambda \\ c_w a F & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{8}$$

Proposition 3 For a $\zeta(x)$ that is expressed in Eq. 6, we observe the following:

1. x^* (i) decreases with c_p , (ii) increases with c_w , a , and F , and (iii) decreases with λ if $c_p < \frac{1}{e} c_w a F \lambda$, increases with λ if $\frac{1}{e} c_w a F \lambda \leq c_p < c_w a F \lambda$, and remains insensitive to λ if $c_p \geq c_w a F \lambda$.
2. U^* (i) increases with c_p , (ii) strictly increases with c_w , a , and F , and (iii) decreases with λ .

Fig. 4 Variation of $\zeta(x)$ as a function of x given $\lambda_1 > \lambda_2 > \lambda_3$



4 Data sources and parameter estimation

To apply our model to different regions of the USA, the various parameter values are obtained from or estimated using past data. In this paper, different states in the USA are chosen as regions of study. The acres of wildfires and Rx in a state, z and x , respectively, are obtained from the NIFC's website for the years 2003–2017 (National Interagency Fire Center 2022). After 2018, NIFC does not provide acres of Rx by state. Data on forest cover, F , in the USA, broken down by state, are obtained from the 2016 Annual Forest Inventory and Analysis Report (USDA Forest Service 2017). The vulnerability of a state to wildfires, $\zeta(x)$, is given by $\frac{z}{F}$. We fit $\zeta(x)$ to an exponential function of the form $\zeta(x) = ae^{-\lambda x}$, as given in Eq. 6. Here, λ represents the estimated impact of Rx in a state, and a represents the estimated baseline vulnerability of a state to wildfires when no fuel management is done.

The cost per acre of Rx, c_p , is different for each region, and is based on a technical report (Cleaves et al. 2000) by the United States Department of Agriculture (USDA). The report presents the results of a survey, conducted in the year of 2000, on the costs of Rx in Regions 1 (Northern), 2 (Rocky Mountain), 3 (Southwestern), 4 (Intermountain), 5 (Pacific Southwest), 6 (Pacific Northwest), 8 (Southern), and 9 (Eastern) of the National Forest System. Data on Region 10 (Alaska) was not reported. The cost of conducting an acre of Rx is given in 1994 US dollars and includes the planning cost. For each region, the cost of conducting an acre of four different types of Rx (slash reduction, management-ignited, prescribed natural fire, and brush, range and grassland fires) and total amount spent on each type is provided. We use this data to find the weighted average cost of all types of Rx in each region, where the weights are given by the acres of each type of Rx conducted. These costs are then adjusted for inflation to 2017 US dollars. While most states are located entirely in one of the given regions, some states are located across multiple regions. The c_p value for each state is given as the c_p value of the region where the state is primarily located. For instance, states like Oregon have a c_p value of 513.65 USD, while Montana has a c_p value of 247.56 USD. States that face very low wildfire risk, such as Maine, have a c_p value of 91.76 USD. One report from Oregon in 2018 indicates that the cost to conduct Rx ranged from 300 to 700 USD per acre (Bennett et al. 2018). This corroborates our estimation of 513.65 USD per acre in Oregon. In summary, the c_p value for a region is given as below, following Eq. 3:

$$c_p = \frac{\sum_{j=1}^4 A_j c_{pj}}{\sum_{j=1}^4 A_j} \times \iota$$

where ι is used to adjust the value for inflation to 2017 USD. As mentioned above, the four types of Rx are (i) slash reduction, (ii) management-ignited, (iii) prescribed natural fire, and (iv) brush, range, and grassland fires.

The Oregon Forest Resources Institute published a report in 2018 detailing the losses caused by wildfires in 2017 in Oregon (Oregon Forest Resources Institute 2018). Many of the losses, such as harmful air quality and cancellation of sporting events at schools, are not monetized. The total loss due to wildfires is given as a sum of all monetized losses, including the cost of fire suppression and tourism losses, thus leading to a modest estimation of the actual monetary loss. The breakdown of the total losses is tabulated in Table 2. The total losses in Oregon in 2017 are divided by the acres of wildfires in Oregon in the same year to obtain an estimation of the c_w value for Oregon. Since the economic damages of a fire increase with the population density of a region (Doerr and Santín 2016), the c_w

Table 2 The breakdown of costs and losses per acre of wildfire, c_w for Oregon in 2017

Reason	2017 US Dollars
Fire suppression	454,000,000
Travel and tourism (direct losses)	3,270,000
Transportation	228,000
Wood products industry	1,000,000
Forests: private resource losses	60,000,000
Total loss (c_w)	518,498,000
Acres of wildfires (z)	714,520
$c_w = \frac{c_{wz}}{z}$ in Oregon	725.65

of Oregon normalized by its population density is used to estimate the c_w values for other states, based on their respective population densities.

5 Results and discussion

5.1 Impact of Rx across US states

From Sect. 3, we know that λ indicates the impact of Rx programs in decreasing the vulnerability of forest area to wildfires. We obtain λ using an exponential regression of past data as given in Eq. 6, where a higher lambda denotes a larger impact per acre of Rx. Since the λ values are estimated via regression, it is crucial that the statistical significance of the λ value is considered as well, in our case, by taking into account the p value indicating λ 's significance. A p value higher than 0.1 indicates that there is no significant evidence from the data that Rx explain any of the variance in wildfires. Our objective here is not to accurately predict wildfires, but to estimate the significance of lambda.

The impacts of existing Rx programs in different states in the USA are compared. Many states such as Alaska (AK), Hawaii (HI), and most Northeast states have very low numbers and acres of Rx conducted which cause inconsequential results. These numbers are due to the relatively low risk of severe (in terms of damage) wildfires in the region. Therefore, we classify US states that burned less than 0.1% of forest area in 2020 as low risk. Table 3 shows the values of λ and p values for the high-risk states, in order of smallest to largest p value. The impacts in these states are calculated from the years 2003 to 2017. Of these 21 analyzed states, five states (Missouri (MO), New Mexico (NM), Oregon (OR), Nevada (NV), and California (CA)) have a p value less than or equal to 0.1, indicating a significant impact of Rx on wildfires. Of these five states, two states (California and Oregon) have a positive λ value, indicating a reduction in wildfires with increasing Rx. The other three states (Missouri, New Mexico, and Nevada) have negative λ values indicating that the Rx conducted are not impactful at reducing wildfires on a large scale. California faces some of the largest losses in the USA each year from crippling wildfires (in 2020, CA burned 13% of forests in wildfires) and conducts Rx primarily for wildfire risk reduction (CA Air Resources Board 2021). On the other hand, Oregon burned 3.8% of their forest area in 2020. As mentioned in Sect. 5.2, Oregon also has an active Rx Council.

Based on the vulnerability to wildfire, λ values, and p values, we classify all US states into four different categories as given below. The classification is also depicted in Fig. 5.

legislation, and was one of the first states to start a Rx council. Thus, states like Florida are not unimpactful; on the contrary, they conduct enough Rx to maintain a healthy ecosystem, although this means that their per-acre impact is low.

3. States with high risk and significant “bad” impact: States in this category, such as Texas, burned more than 0.1% of forest area in 2020, have p values less than 0.1, and negative λ values. These states’ negative λ values indicate that larger Rx programs in these states have a significant positive correlation with larger wildfire seasons. One possible explanation for this could be that these states increase their conducted Rx when there is a higher risk of wildfires, but the Rx conducted are not impactful.
4. States with high risk and significant “good” impact: States in this category (California and Oregon) burned more than 0.1% of forest area in 2020, have p values less than 0.1, and positive λ values. Past works claim that states in the western USA do not conduct as much Rx as they should (Kolden 2019). Our results also indicate the same in Sect. 5.2.2 where, using our optimization model, we show that both California and Oregon should conduct a lot more Rx than they currently do.

5.2 Case study in Oregon

In this section, we implement the models developed in this paper to the State of Oregon. Some of the data used are given in Table 4. Past research works, such as Kim et al. (2009), have often chosen Oregon as a study site due to the prevalence of Rx there and the support for Rx from agencies such as the Oregon Rx Council (Oregon Prescribed Fire Council 2022). Oregon is one of the few states that has a high risk for wildfires, but also consistently uses Rx as a mitigation mechanism (Daugherty and Fried 2007). The aforementioned council has pushed for policy changes, fostered discussion, educated the

Table 4 Values of the acres of wildfires, acres of prescribed fires, acres of forest, and vulnerability of a region to wildfires, z , x , F , and ζ , respectively, in the State of Oregon from 2003 to 2017

Year	z (acres)	x (acres)	F (acres)	ζ
2003	160,441	102,236	29,804,000	0.0054
2004	31,220	138,880	29,804,000	0.0010
2005	192,081	97,065	29,804,000	0.0064
2006	548,870	112,949	29,804,000	0.0184
2007	648,046	110,876	29,804,000	0.0217
2008	140,221	112,450	29,804,000	0.0047
2009	100,668	130,654	29,804,000	0.0034
2010	93,731	114,716	29,804,000	0.0031
2011	285,712	73,855	29,804,000	0.0096
2012	1,256,049	60,838	29,804,000	0.0421
2013	350,786	74,684	29,804,000	0.0118
2014	984,629	88,887	29,804,000	0.0330
2015	685,809	87,973	29,804,000	0.0230
2016	219,509	84,442	29,804,000	0.0074
2017	714,520	48,077	29,804,000	0.0240

public, and provided assistance and training as needed. In January 2019, the State of Oregon approved New Air Quality Rules to allow for more Rx (Burns 2019).

5.2.1 Impact of Rx in Oregon

We fit an exponential relationship between ζ for a given year and x in the previous year in the State of Oregon for the years 2004–2017. This is done because Oregon is one of the states that conducts Rx primarily in the fall, after the wildfire season. The ζ values are obtained by dividing the acres of wildfires, z , by the forest cover, F , in Oregon. For this time period, we obtain the relationship: $\zeta = 0.124e^{-2.55 \times 10^{-5}x}$. Here, 12.4% of Oregon’s forests are estimated to burn with no Rx, and one acre of Rx reduces the vulnerability to $0.124e^{-2.55 \times 10^{-5}}$. We know that this Rx program has a significant impact on wildfires because this relationship has a p value of 0.05.

5.2.2 Optimization

The least-cost optimization model from Sect. 2 is depicted graphically for the year 2017 in Oregon in Fig. 6. We assume that the cost of Rx increases linearly with an increase in the acres of Rx, as shown in Fig. 6. Vulnerability to wildfires decreases exponentially with an increase in the acres of Rx. Applying the least-cost optimization model to Oregon from 2012 to 2017, we see that the optimal acres of Rx to be conducted are significantly higher than the actual acres of Rx each year, as shown in Fig. 7. In this case, the model prescribes conducting more Rx to obtain a significant decrease in the losses due to wildfires as shown in Fig. 8. It is of note here that the exponential fit for the year 2012 is not statistically significant, while the rest of the years are. From 2013 to 2017, we observe an average recommended increase of 109,841 acres of Rx (143% increase) as shown in Fig. 7. The parameters such as λ and a for each year are estimated using data from the previous 8 years. These numbers are chosen to account for the bias-variance trade-off (Brownlee 2016). A larger number of years is not chosen so as to avoid overfitting and take into account only recent history while estimating parameters. On the other hand, a smaller number is not chosen so

Fig. 6 Total cost and loss as functions of investment in OR for the year 2017

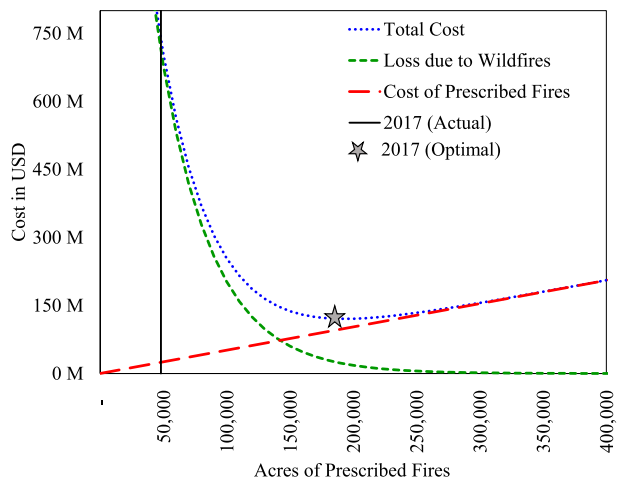


Fig. 7 A comparison of actual versus optimal acres of Rx for OR (2012–2017)

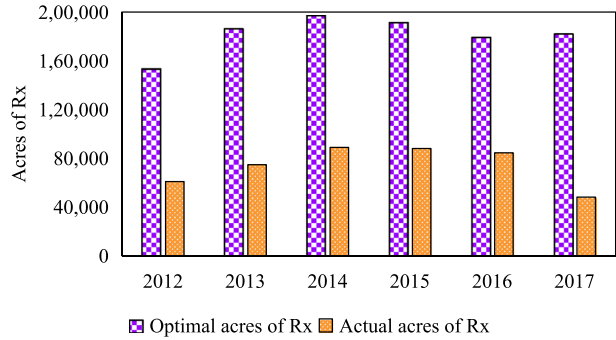
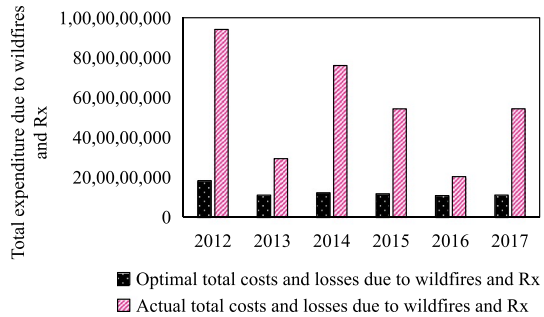


Fig. 8 A comparison of actual versus optimal total costs and losses for OR (2012–2017)



as to provide a sufficient number of data points to obtain an accurate estimate of parameters and to avoid high bias.

From Fig. 8, it is observed that the optimal total costs and losses in Oregon are less than the actual total costs and losses each year. From 2012 to 2017, there is an average decrease in total costs and losses of 355 million USD which leads to, on average, an optimal value which is 24% of the actual costs and losses each year. A similar analysis is done in California from the years 2012–2017. We only obtain statistically significant results for the year 2017 in CA, where the model suggested that almost 170,000 more acres should have been burned to reduce the total costs and losses by over 5 billion USD.

5.2.3 Sensitivity analysis

We study how x^* and U^* in Oregon are sensitive to changes in the parameters c_p , c_w , a , F , and λ . We use the following baseline values of these parameters for the year 2017: $c_p = \$513.65$, $c_w = \$725.66$, $a = 0.19$, $F = 29,804,000$ acres, and $\lambda = 3.02 \times 10^{-5}$.

Figure 9a illustrates the changes in x^* and U^* with respect to the parameter c_p . It is observed that with an increase in c_p , x^* shows a decreasing trend, while U^* increases. A lower value of c_p motivates the landowners and forest managers to conduct more Rx in order to mitigate the risks of wildfires with a simultaneous reduction in the total costs and losses. Hence, in order to encourage the practice of conducting Rx, it is important to provide financial incentives so as to keep the value of c_p in an affordable range.

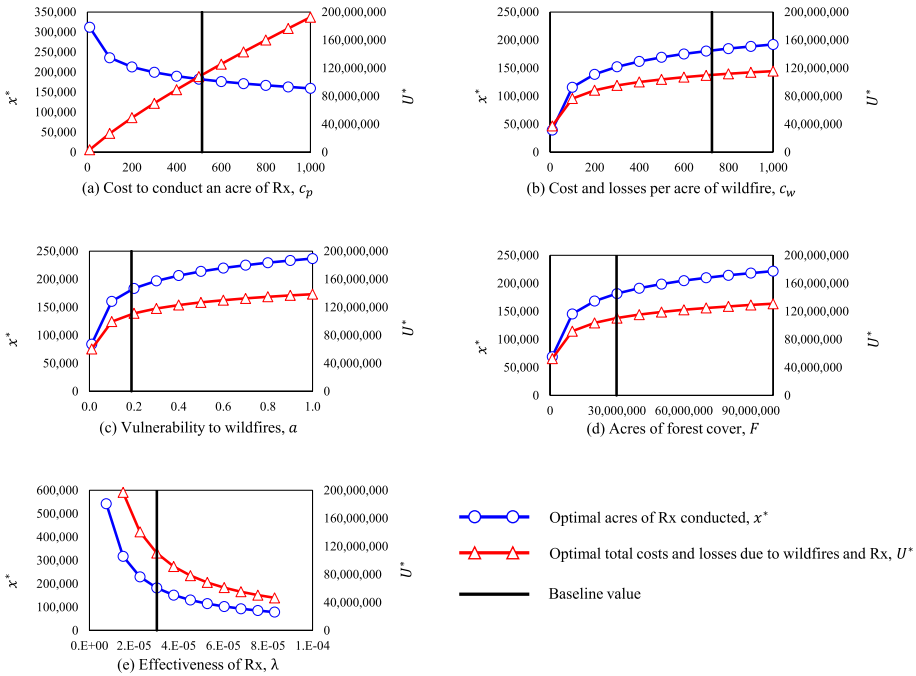


Fig. 9 Sensitivity analysis of x^* and U^* with respect to parameters c_p , c_w , a , F , and λ

Figure 9b illustrates the changes in x^* and U^* with respect to the parameter c_w . It is observed that with an increase in c_w , both x^* and U^* show an increasing trend. The higher the loss per acre of wildfire, the higher the acres of Rx that should be conducted by landowners and forest managers. As a result of higher losses and conducting more Rx, the total costs and losses also increase.

Figure 9c illustrates the changes in x^* and U^* with respect to the parameter a which can be conceptualized as a measure of vulnerability to wildfire. It is observed that with an increase in a , both x^* and U^* show an increasing trend. For higher values of a , more emphasis should be given to conducting a large number of acres of Rx to decrease risk. However, as a result of higher losses and conducting more Rx, the total costs and losses also increase.

Figure 9d illustrates the changes in x^* and U^* with respect to the parameter F . It is observed that with an increase in F , both x^* and U^* show an increasing trend. For higher values of F , the expected acres of wildfires in a region also increase. This risk can be mitigated by conducting more acres of Rx, which in turn will result in an increase in the total costs and losses.

Figure 9e illustrates the changes in x^* and U^* with respect to the parameter λ . It is observed that with an increase in λ , both x^* and U^* show a decreasing trend. A high value of λ indicates a high impact per acre of Rx in mitigating wildfire risk. Hence, fewer acres of Rx can be conducted to achieve the same level of risk mitigation. Because fewer acres of Rx are conducted, the total costs and losses decrease. It is vital to conduct Rx optimally to increase their impact, as this will decrease total costs and losses.

In our study, we observe that an increase in the parameter c_p leads to a decreasing trend in x^* , while U^* increases. Similarly, an increase in c_w , a , F , or λ shows an

increasing trend in both x^* and U^* . Our findings suggest that financial incentives could encourage the practice of conducting Rx to mitigate the risks of wildfires and to keep the value of c_p in an affordable range. We also emphasize the importance of conducting Rx optimally to increase their impact, as this will decrease total costs and losses.

6 Recommendations for US states

All US states were classified into four different categories in Sect. 5.1, as depicted in Fig. 5. In this section, we recommend future steps to take for states in each category.

1. States with low risk: While these states do not require immediate changes due to their low risk of wildfires, wildfire risks are not necessarily confined to the west coast and it is recommended that these states be fire pro-active (U.S. Fish and Wildlife Service 2008).
2. States with high risk, no perceived impact:
 - (a) Many of these states struggle with Rx that seems to be changing nothing. Works such as Palaiologou et al. (2020) have suggested that random or scattered fuel treatment is not as effective as fuel treatment that is spatially optimized. Additionally, Rx is often not conducted in the regions that could have the highest impact on wildfire losses, such as WUI regions, due to the correspondingly high potential liabilities of Rx in these regions (Miller et al. 2020). Such states could increase their impact on wildfires by integrating better Rx planning strategies and by changing regulations that prevent Rx from being conducted in the regions that need it the most.
 - (b) A small subsection of states in the southeast, on the other hand, burn a lot. Florida and Georgia alone burned almost 3.5 million acres of Rx in 2017 and consistently burn more Rx than any other state. These states reap many benefits from the Rx conducted, as evidenced by their relatively smaller wildfire-related losses. States like these continue a healthy fire-dependent ecosystem that was historically practiced by indigenous people. In these states, disastrous wildfires are naturally kept at bay, as is ideal (Buono 2020).
3. States with high risk and significant “bad” impact: Three states fall into this category: Missouri, New Mexico, and Nevada. Like states that fall in category 2(a), these states could benefit from changing the way Rx are conducted (Palaiologou et al. 2020).
4. States with high risk and significant “good” impact: California and Oregon fall into this category. While these states already conduct Rx that reduce wildfire risk, they could increase the overall impact on wildfires by increasing the scale of their Rx programs. The optimal scale of Rx programs that should be conducted by these states to reduce the overall costs and losses of wildfire is studied in Sect. 3.

Overall, based on data up to 2017, we observe that most Rx programs in the USA are lacking and have not had a significant impact on wildfire risk. These results could feed into misconceptions that Rx are a risky, expensive, and ineffective practice to introduce to fire-ravaged ecosystems. On the contrary, our results show that Rx programs can be impactful when carefully designed and planned, as in Oregon and California. Other high-risk states

are recommended to take action to improve their Rx programs by (i) encouraging the formation and growth of Prescribed Burn Associations (PBA) (Toledo et al. 2014) and Rx Councils to both plan Rx in areas that need it the most and increase the acreage of Rx, (ii) maintaining cohesive and detailed Rx logs that can be used to diagnose and improve Rx programs, (iii) changing laws and regulations that prevent Rx, and (iv) providing resources, training, and aid to organizations and landowners to conduct Rx. States that already conduct impactful Rx programs should focus on expanding their efforts and increasing acreage. States like Florida and Georgia, where healthy fire-dependent ecosystems are maintained, should aid other states with Rx training and resources in addition to continuing their Rx programs.

7 Conclusions and future research directions

Wildfires in the USA have been increasing in intensity over the past decade. In the western USA, the 2020 wildfire season burned over 10 million acres, destroyed over 13,000 buildings, caused 46 fatalities, and had an estimated cost greater than 19 billion USD. This unfortunate trend can be observed globally as well. For example, the 2019–2020 wildfire season in Australia burned over 46 million acres and destroyed over 9000 buildings in roughly 12 months. Fuel management techniques, such as Rx, are effective at decreasing the intensity of wildfires and are carried out throughout the world.

In past works, Rx have been proved to be effective at reducing both the risk as well as magnitude of wildfires, in addition to having a host of other benefits. Research has also been done optimizing the pattern, location, and amount of Rx or fuel management. While we know that Rx works, and that Rx can be impactful depending on how they are conducted, the impact of Rx programs on a larger scale has not been studied. For example, not burning Rx at the right places or right times could significantly reduce their impact. To the best of our knowledge, a comparative analysis of the impact of Rx programs in different states in the USA has not been conducted. We address these gaps by (i) determining the impact of the Rx program in a region using data analysis, (ii) developing an optimization model that finds the optimal scale of impactful Rx programs, and (iii) comparing the Rx programs of different states in the USA.

We analyze and compare the impact of an Rx program in a region by conducting an exponential regression of the acres of wildfires with respect to acres of Rx. An optimization model to minimize the total costs and losses of wildfires, including the costs and losses of wildfires and the costs of Rx, is developed using the results of the exponential regression. It is discovered that California and Oregon conduct the only significantly impactful Rx among analyzed states. 19 of the 21 analyzed states (such as Colorado) do not seem to have Rx programs that have significant impact on their wildfire risk. However, some of these states (such as Florida) do not seem to have significant impact per acre while actually conducting very successful and expansive Rx programs. This is because these states burn many millions of acres, like was done by indigenous peoples, where acres of Rx are not burned exclusively for wildfire risk reduction, but to maintain a healthier ecosystem. Other states (such as Hawaii) are not analyzed because they do not have significant wildfire risk. Each year from 2013 to 2017, Oregon could save an average of 355 million USD by increasing the Rx an average of 109,841 acres. In 2017, in California, savings of over 5 billion USD are estimated if an additional 170,000 acres were burned in the state.

Our work can be expanded to locations outside the USA given access to data on Rx and wildfires. Our optimization model can be extended to consider the effects of past Rx and wildfires on fire risk in order to gain deeper insights into the optimal amount of Rx to conduct in a region. In the future, these models can also be applied to study other fuel management techniques such as logging. Future research can also extend the current study to model public-private partnerships for developing collaborative relationships to encourage the use of Rx when necessary. In addition, incorporating the risk preferences of stakeholders (Hunt et al. 2022) can provide important insights into decision-making for fuel management.

Appendix

Proof for Proposition 1

In order to solve for x^* and U^* , we take the derivative of $U(x)$ with respect to x and set it equal to zero to solve for the optimal acres of Rx, x^* , as follows:

$$\frac{dU(x)}{dx} = c_w \mathcal{G}(x)F + c_p = 0 \tag{9}$$

$$x^* = \mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right) \tag{10}$$

where $\mathcal{G}(x) = \zeta'(x)$. Since we assume that $\zeta'(x)$ is strictly increasing for $x \geq 0$, therefore \mathcal{G}^{-1} exists. The above value of x^* warrants a minima for $U(x)$ since:

$$\frac{d^2U(x)}{dx^2} = c_w \mathcal{G}'(x)F = c_w \zeta''(x)F > 0, \because c_w, F, \zeta''(x) > 0 \tag{11}$$

For $x^* > 0$, $\mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right) > 0$, which implies $c_p < -\mathcal{G}(0)c_w F$. On the other hand, $x^* = 0$ if $c_p \geq -\mathcal{G}(0)c_w F$. Now, we substitute the value of x^* in Eq. 2 to get U^* as shown below:

$$U^* = U(x^*) = \begin{cases} c_w \zeta\left(\mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right)\right)F + c_p \mathcal{G}^{-1}\left(-\frac{c_p}{c_w F}\right) & \text{if } c_p < -\mathcal{G}(0)c_w F \\ \zeta(0)c_w F & \text{if } c_p \geq -\mathcal{G}(0)c_w F \end{cases} \tag{12}$$

Proof for Proposition 2

We consider $\zeta(x) = ae^{-\lambda x}$, where $a, \lambda > 0$. For $x \geq 0$, we observe that the exponential decay form of $\zeta(x)$ is continuous and twice differentiable and it also satisfies the assumptions as shown below:

$$\zeta(x) > 0 \tag{13}$$

$$\zeta'(x) = -a\lambda e^{-\lambda x} = -\lambda\zeta(x) < 0 \tag{14}$$

$$\zeta''(x) = a\lambda^2 e^{-\lambda x} = \lambda^2 \zeta(x) > 0 \tag{15}$$

In order to solve for x^* and U^* , we take the derivative of $U(x)$ with respect to x and set it equal to zero to solve for the optimal number of Rx, x^* as follows:

$$\frac{dU(x)}{dx} = -\lambda C_w a F e^{-\lambda x} + c_p = 0 \tag{16}$$

$$x^* = -\frac{1}{\lambda} \ln\left(\frac{c_p}{c_w a F \lambda}\right) \tag{17}$$

For x^* to be positive, $-\frac{1}{\lambda} \ln\left(\frac{c_p}{c_w a F \lambda}\right) > 0$, which implies $c_p < c_w a F \lambda$. On the other hand, $x^* = 0$ if $-\frac{1}{\lambda} \ln\left(\frac{c_p}{c_w a F \lambda}\right) \leq 0$, which implies $c_p \geq c_w a F \lambda$. The above value of x^* warrants a minima for $U(x)$ since:

$$\frac{d^2 U}{dx^2} = \lambda^2 C_w a F e^{-\lambda x} > 0, \because c_w, a, F, \lambda > 0 \tag{18}$$

Now, we substitute the value of x^* in Eq. 2 to get U^* as follows:

$$U^* = U(x^*) = \begin{cases} \frac{c_p}{\lambda} \left(1 - \ln\left(\frac{c_p}{c_w a F \lambda}\right)\right) & \text{if } c_p < c_w a F \lambda \\ c_w a F & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{19}$$

Proof for Proposition 3

The sensitivity of x^* and U^* as a function of c_p, c_w, a, F, λ are proved as follows:

For c_p ,

$$\frac{\partial x^*}{\partial c_p} = \begin{cases} -\frac{1}{\lambda c_p} < 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{20}$$

$$\frac{\partial U^*}{\partial c_p} = \begin{cases} -\frac{1}{\lambda} \ln\left(\frac{c_p}{c_w a F \lambda}\right) > 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{21}$$

For c_w ,

$$\frac{\partial x^*}{\partial c_w} = \begin{cases} \frac{1}{\lambda c_w} > 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{22}$$

$$\frac{\partial U^*}{\partial c_w} = \begin{cases} \frac{c_p}{\lambda c_w} > 0 & \text{if } c_p < c_w a F \lambda \\ a F > 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \tag{23}$$

For a ,

$$\frac{\partial x^*}{\partial a} = \begin{cases} \frac{1}{\lambda a} > 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \quad (24)$$

$$\frac{\partial U^*}{\partial a} = \begin{cases} \frac{c_p}{\lambda a} > 0 & \text{if } c_p < c_w a F \lambda \\ c_w F > 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \quad (25)$$

For F ,

$$\frac{\partial x^*}{\partial F} = \begin{cases} \frac{1}{\lambda F} > 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \quad (26)$$

$$\frac{\partial U^*}{\partial F} = \begin{cases} \frac{c_p}{\lambda F} > 0 & \text{if } c_p < c_w a F \lambda \\ c_w a > 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \quad (27)$$

For λ ,

$$\frac{\partial x^*}{\partial \lambda} = \begin{cases} \frac{1}{\lambda^2} \left(\ln \left(\frac{c_p}{c_w a F \lambda} \right) + 1 \right) & \begin{cases} < 0 & \text{if } c_p < \frac{1}{e} c_w a F \lambda \\ > 0 & \text{if } \frac{1}{e} c_w a F \lambda \leq c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \\ 0 & \end{cases} \quad (28)$$

$$\frac{\partial U^*}{\partial \lambda} = \begin{cases} \frac{c_p}{\lambda^2} \ln \left(\frac{c_p}{c_w a F \lambda} \right) < 0 & \text{if } c_p < c_w a F \lambda \\ 0 & \text{if } c_p \geq c_w a F \lambda \end{cases} \quad (29)$$

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Declarations

Conflict of interest The authors have no conflict of interest (financial or non-financial) to disclose.

Human participants and/or animals This research does not involve human participants and/or animals.

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